

Math Club Problem Set #6 Venn Diagrams and Counting Principles

1. A school only offers Spanish and French as alternative languages. There are 40 students enrolled in at least one of the classes. If 28 students are in Spanish and 23 are in French, how many students are taking both languages?
2. How many 6 digit numbers start or end with an even digit?
3. There are 47 dogs at a pound. All of them are big or very hairy. 30 are big, 42 are hairy. How many of the dogs are big, very hairy dogs?
4. Suppose that 80% of households own a DVD player and that 70% of households own a computer. What is the range of possible percentages of households that own both?

5. How many 9 digit numbers have the property that the product of their first and last digit is even?

6. A school with 100 students offers French and Spanish as its language courses. Twice as many students are in French than Spanish. Three times as many students are in both classes as are in neither classes. The number of students in both classes is even, and fewer than 10 students are in neither class. How many students are taking Spanish?

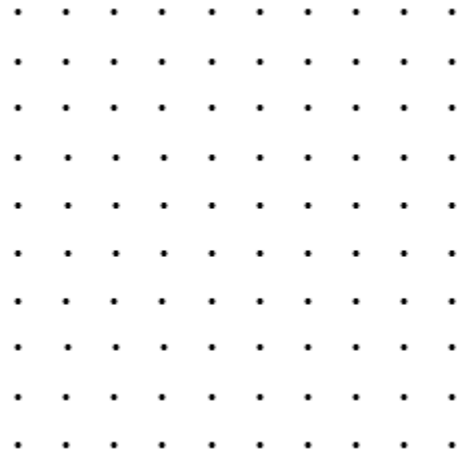
7. How many positive integers less than 180 are relatively prime to 180?

8. A family has 3 boys and 3 girls. In how many ways can the 6 kids be seated in a row of 6 chairs, so that the boys aren't all seated together and the girls aren't all seated together?

9. How many positive integers less than 100,000 are neither squares nor cubes?
10. Is it possible that among a group of 20 students, 15 of them play tennis, 12 of them badminton, and 6 of them play both? Explain:
11. When I go to work, there is a 20% probability that I will forget my keys, 30% probability that I will forget my wallet. If there's a 5% chance that I forget both, what is the probability that I arrive at work with both my keys and wallet?
12. How many 5 digit sequences have a digit that appears at least 3 times? (For instance, 03005 and 22922 are examples of such sequences.)

13. How many 10 digit binary numbers start with 2 ones or end with 2 ones (or both)?

14. A square array of dots with 10 rows and 10 columns is given. Each dot is coloured either blue or red. Whenever two dots of the same colour are adjacent in the same row or column, they are joined by a line segment of the same colour as the dots. If they are adjacent but of different colours, they are then joined by a green line segment. In total, there are 52 red dots. There are 2 red dots at corners with an additional 16 red dots on the edges of the array. The remainder of the red dots are inside the array. There are 98 green line segments. How many blue line segments are there?



15. How many numbers less than 1000 are not a multiple of 2, 3, 5, or 7?

1. 11 students	2. 650,000	3. 25 dogs	4. 50%-70%
5. 650million	6. 36 students	7. 48 numbers	8. 504 ways
9. 99,643	10. Impossible	11. 55%	12. 8560
13. 320 numbers	14. 37 lines	15.	